This activity is something of a jigsaw exercise. Collect data with your group, and then summarize your task and the data you found on a neat and attractive poster to share out.

**Continued Fractions**

Express the following numbers as fractions in lowest terms.

\[ A_0 = 1 \]

\[ A_1 = 1 + \frac{1}{2} \]

\[ A_2 = 1 + \frac{1}{2 + \frac{1}{2}} \]

\[ A_3 = 1 + \frac{1}{2 + \frac{1}{2 + \frac{1}{2}}} \]

\[ A_4 = 1 + \frac{1}{2 + \frac{1}{2 + \frac{1}{2 + \frac{1}{2}}}} \]

Japheth Wood <japheth@nymathcircle.org>    July 16, 2012
This activity is something of a jigsaw exercise. Collect data with your group, and then summarize your task and the data you found on a neat and attractive poster to share out.

**Two Intertwined Recursive Sequences**

The two sequences $P_1, P_2, P_3, \ldots$ and $Q_1, Q_2, Q_3, \ldots$ are defined as follows. Compute the first five terms in each sequence.

\[
\begin{align*}
P_1 &= 1, \quad Q_1 = 1 \\
P_2 &= P_1 + 2Q_1, \quad Q_2 = P_1 + Q_1 \\
P_3 &= P_2 + 2Q_2, \quad Q_3 = P_2 + Q_2 \\
P_4 &= P_3 + 2Q_3, \quad Q_4 = P_3 + Q_3 \\
P_5 &= P_4 + 2Q_4, \quad Q_5 = P_4 + Q_4
\end{align*}
\]
This activity is something of a jigsaw exercise. Collect data with your group, and then summarize your task and the data you found on a neat and attractive poster to share out.

Matrix Powers

Let \( C = \begin{pmatrix} 1 & 1 \\ 2 & 1 \end{pmatrix} \) and compute the following:

\[
C^0 =
\]

\[
C^1 =
\]

\[
C^2 =
\]

\[
C^3 =
\]

\[
C^4 =
\]

\[
C^5 =
\]
This activity is something of a jigsaw exercise. Collect data with your group, and then summarize your task and the data you found on a neat and attractive poster to share out.

**Babylonian Fractions**

Evaluate the first 5 terms of the recursively defined sequence (as fractions in lowest terms):

\[
S_0 = 1
\]

\[
S_1 = \frac{1}{2} \left( S_0 + \frac{2}{S_0} \right)
\]

\[
S_2 = \frac{1}{2} \left( S_1 + \frac{2}{S_1} \right)
\]

\[
S_3 = \frac{1}{2} \left( S_2 + \frac{2}{S_2} \right)
\]

\[
S_4 = \frac{1}{2} \left( S_3 + \frac{2}{S_3} \right)
\]

\[
S_5 = \frac{1}{2} \left( S_4 + \frac{2}{S_4} \right)
\]

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This activity is something of a jigsaw exercise. Collect data with your group, and then summarize your task and the data you found on a neat and attractive poster to share out.

**Counting Problems**

How many ways can you tile a $1 \times n$ board with $1 \times 1$ squares and $1 \times 2$ dominoes, according to the following rules?

$T(n)$: Each space of the $1 \times n$ board can be covered by either a domino or by squares. If covered by squares, then the space can have a stack of either one or two squares.

Compute $T(1), T(2), T(3), T(4), \text{ and } T(5)$.

$B(n)$: Each space of the $1 \times n$ board can be covered by either a domino or by squares. If covered by squares, then the space can have a stack of either one or two squares. Except for the first space, which (if not covered by a domino) can have a stack of only one square on it.

Compute $B(1), B(2), B(3), B(4), B(5)$. 
This activity is something of a jigsaw exercise. Collect data with your group, and then summarize your task and the data you found on a neat and attractive poster to share out.

**Algebraic Number Theory**

Compute (and simplify) positive integer powers of $1+\sqrt{2}$.

$(1 + \sqrt{2})^0 =$

$(1 + \sqrt{2})^1 =$

$(1 + \sqrt{2})^2 =$

$(1 + \sqrt{2})^3 =$

$(1 + \sqrt{2})^4 =$

$(1 + \sqrt{2})^5 =$

Japheth Wood <japheth@nymathcircle.org>  
July 16, 2012
Continued Fractions

Express the following numbers as fractions in lowest terms.

\[ \begin{align*}
A_0 &= 1 \\
A_1 &= 1 + \frac{1}{2} \\
A_2 &= 1 + \frac{1}{2 + \frac{1}{2}} \\
A_3 &= 1 + \frac{1}{2 + \frac{1}{2 + \frac{1}{2}}} \\
A_4 &= 1 + \frac{1}{2 + \frac{1}{2 + \frac{1}{2 + \frac{1}{2}}}}
\end{align*} \]

Two Intertwined Recursive Sequences

The two sequences \( P_1, P_2, P_3, \ldots \) and \( Q_1, Q_2, Q_3, \ldots \) are defined as follows. Compute the first five terms in each sequence.

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P_5 &= P_4 + 2Q_4, \quad Q_5 = P_4 + Q_4
\end{align*} \]

Matrix Powers

Let \( C = \begin{pmatrix} 1 & 1 \\ 2 & 1 \end{pmatrix} \) and compute the following:

\[ \begin{align*}
C^0 = & \quad C^1 = \quad C^2 = \quad C^3 = \quad C^4 = \quad C^5 = 
\end{align*} \]
Babylonian Fractions

Evaluate the first 5 terms of the recursively defined sequence (as fractions in lowest terms):

\[ S_0 = 1 \]
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\[ S_3 = \frac{1}{2} \left( S_2 + \frac{2}{S_2} \right) \]
\[ S_4 = \frac{1}{2} \left( S_3 + \frac{2}{S_3} \right) \]
\[ S_5 = \frac{1}{2} \left( S_4 + \frac{2}{S_4} \right) \]

Counting Problems

How many ways can you tile a 1 \times n board with 1 \times 1 squares and 1 \times 2 dominoes, according to the following rules?

\[ T(n) : \] Each space of the 1 \times n board can be covered by either a domino or by squares. If covered by squares, then the space can have a stack of either one or two squares.
Compute \( T(1), T(2), T(3), T(4), \) and \( T(5) \).

\[ B(n) : \] Each space of the 1 \times n board can be covered by either a domino or by squares. If covered by squares, then the space can have a stack of either one or two squares. Except for the first space, which (if not covered by a domino) can have a stack of only one square on it.
Compute \( B(1), B(2), B(3), B(4), B(5) \).

Algebraic Number Theory

Compute (and simplify) positive integer powers of \( 1 + \sqrt{2} \).

\[ (1 + \sqrt{2})^0 = \]
\[ (1 + \sqrt{2})^1 = \]
\[ (1 + \sqrt{2})^2 = \]
\[ (1 + \sqrt{2})^3 = \]
\[ (1 + \sqrt{2})^4 = \]
\[ (1 + \sqrt{2})^5 = \]