Questions

(1) Starting at the third “1” along either of the outer diagonals (left or right), consider the diagonal of numbers 1, 3, 6, 10, ... What can you say about these numbers? Is there a pattern to their growth?

(2) Starting at the fourth “1” along either of the outer diagonals, consider the diagonal that looks like 1, 4, 10, 20, ... What can you say about these numbers? Is there a pattern to their growth? What, if anything, do they have in common with the numbers from the previous question?

(3) The numbers from the previous two items are called triangular numbers and tetrahedral numbers, respectively. Can you figure out why?

(4) Pick a row and add up the numbers in it. Try this for other rows and see if you can spot a pattern. Can you give a reason for why this pattern occurs?

(5) Pick a row and change the signs on the numbers so that half of them are negative and with the signs alternating +, −, +, −, +... and then add up these numbers. What do you get? Can you figure out why this happens?

(6) Start at any 1 on the outer right diagonal, and look at the diagonal that juts to the left from that 1. Follow this leftward-pointing diagonal out as many entries as you like and come to a stop. Now consider the number that is one row down and to the right of where you stopped. How does this number relate to the list of numbers on leftward-pointing diagonal path you took to get here? Can you come up with a rule for this? Can you say why the rule always works?

(7) Is there anything that distinguishes rows that have a prime number after the first 1 from rows that have a composite number after the first 1?

(8) Draw a circle. Now draw two points on this circle (space the points roughly evenly away from each other on the circle). How many straight line segments can you draw between these two points?

   Okay, that was dumb. Try it with three points, roughly evenly spaced apart on the circle. How many straight line segments can you draw connecting pairs of points? How many triangles can you draw connecting these points?

   Okay, try it with four points now. How many straight line segments connect pairs of points? How many triangles connect triples of points? How many quadrilaterals can you draw?
Okay, take this up a notch to five points. What patterns can you see in this process? (Go to six or seven points if you like.)

(9) (related to the previous item) Consider the collection of four letters $A, B, C,$ and $D$. How many different ways are there to choose just one of these letters? How many different ways are there to choose just two (notice: I would consider the choice $AB$ to be the same as the choice $BA$, so that order doesn’t matter)? How many different ways are there to choose three letters? What about choosing four letters?

Now try this with five letters $A, B, C, D,$ and $E$. How many different ways can you choose a group of three letters from this collection (again, where order does not matter)? Two letters? Four letters? One letter? Five letters? No letters? What do you notice about these numbers?

If $k$ and $n$ are non-negative integers and $k \leq n$, can you come up with a rule for determining how many ways you can choose $k$ items from a group of $n$ items? (This quantity is typically denoted by the symbol $\binom{n}{k}$. So, for example, $\binom{4}{2} = 6$, because there are 6 ways to choose a pair of objects from a collection of 4 objects).

(10) Take a look at the picture of Pascal’s triangle modulo 2. This means that every odd number in the triangle is replaced by a light dot and every even number is replaced by a dark dot. Which rows have all odd numbers (that is, all light dots)?

We can ask this same question again, only this time we alter the triangle by making an entry of the triangle a light dot if its remainder after dividing by 3 is nonzero and make it a dark dot if 3 divides evenly into it. Look at the corresponding picture of Pascal’s triangle modulo 3 and try to find a pattern to which rows are all light dots. Try this with other numbers (pictures are included). Keep in mind that your answers may differ depending on whether or not the modulo number you choose is prime or composite.

**Some Web References**